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Richards Growth Model and Multi-Metaheuristic Optimization for Precision Irrigation and Fertilization Scheduling: An Extension of the Becker-Zohdi Framework

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Abstract

Agricultural input costs are rising, and weather patterns are becoming more variable. These trends create a need for more effective methods for allocating water and fertiliser in farming. This paper extends the crop model developed by Becker and Zohdi [1]. We replace the logistic growth equation with the Richards model. The Richards model has a shape parameter that allows asymmetric growth. We also compare five metaheuristic algorithms. These are Genetic Algorithm (GA), Particle Swarm Optimisation (PSO), Differential Evolution (DE), Grey Wolf Optimiser (GWO), and a Hybrid GA-PSO. The study uses corn production data from Iowa. We test the methods on 21 stochastic weather scenarios. The Richards model captures the vegetative and reproductive phases of corn more accurately than the logistic model. Among the algorithms, GWO gives the highest average revenue. It achieves $903 per acre. The GWO result is higher than GA ($876/acre), DE ($884/acre), PSO ($891/acre), and Hybrid GA-PSO ($898/acre). Such a result represents a 7% improvement over the original GA-logistic result of $842/acre. Statistical tests confirm the superiority of GWO with 95% confidence. The main findings are that asymmetric growth modelling improves optimisation results, especially under stress conditions. GWO provides a good balance between exploration and exploitation. This work provides a benchmark for metaheuristic performance in precision agriculture. It also shows that the Richards model is a better choice than the logistic model for this type of problem.

Keywords: Precision agriculture, Richards growth model, Metaheuristic algorithms, Grey wolf optimiser, Irrigation scheduling, Crop optimisation.

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1|Introduction

Farming in the United States faces several pressures. Production costs continue to rise. Commodity prices remain unstable [2]. Farmers are looking for computational tools to improve their resource use. These tools do not require large capital investments. Recent work on digital twins [3], [4] and machine learning [5], [6] shows the potential of computational methods in precision agriculture.

Crop growth models have evolved over time. Early models used simple empirical formulas. Later models became more complex. The logistic growth model [7] is still common. It is easy to interpret. It describes growth under resource limits. However, Becker and Zohdi [1] noted a limitation. The logistic model assumes symmetric growth around the inflection point. This assumption may not be accurate for crops like corn. Corn has clear vegetative and reproductive phases.

Various optimisation techniques have been applied to irrigation and fertiliser management. These include linear programming [8], dynamic programming [9], and metaheuristic algorithms [10]. Genetic Algorithms (GAs) are popular in this domain. They handle nonlinear problems well. They do not require gradient information [11]. However, a systematic comparison of multiple metaheuristic algorithms on the same crop growth optimisation problem is lacking in the literature.

Becker and Zohdi [1] made a significant contribution. They built a coupled ODE crop model. The model includes delayed nutrient absorption through FIR convolution. It also includes cumulative stress tracking through EMA filtering. Their GA improved average revenue by 35% compared to farmer practices. The approach was tested across 21 weather scenarios. In their limitations section, they state: "The logistic *Eq. (1)* assumes symmetric growth around the inflection point. Richards growth generalises this with a shape parameter v"

They also mention that further work could examine higher temporal resolution in the GA. Their optimisation used only one metaheuristic approach. It is therefore unclear whether other algorithms could give better results.

This study has two main objectives. First, we replace the logistic model with the asymmetric Richards model. We add shape parameters for each of the five state variables. These are calibrated with physiological data from our co-author in plant biology. Second, we compare five metaheuristic algorithms: GA, Particle Swarm Optimisation (PSO), Differential Evolution (DE), Grey Wolf Optimiser (GWO), and a Hybrid GA-PSO. We use the same objective function, weather scenarios, and experimental setup as Becker and Zohdi [1]. Such consistency ensures a fair comparison.

The paper is organised as follows. Section 2 reviews the relevant literature. Section 3 describes the extension of the Richards model. Section 4 explains the stochastic weather scenarios. These follow the same procedure as Becker and Zohdi [1]. Section 5 presents the five algorithms. Section 6 gives the experimental results and statistical analysis. Section 7 discusses the findings and limitations. Section 8 provides practical recommendations.

2|Literature Review

2.1|Crop Growth Models

Crop growth modeling has progressed from simple empirical equations to complex mechanistic simulations. These simulations integrate physiological, environmental, and management factors.

Early empirical models used statistical relationships between environmental variables and final yield. These models were simple to apply. However, they lacked a mechanistic basis. They could not predict responses to new conditions or management practices reliably. The logistic growth model [7] was a significant advancement. It provided a mathematical description of resource-limited growth. The logistic equation assumes that growth rate depends on both current size and remaining capacity. Such an assumption produces a symmetric S-shaped curve. The model is widely used because it is interpretable and computationally simple.

Process-based crop models appeared in the 1980s and 1990s. These included comprehensive simulation platforms. The DSSAT model [12] integrates soil, weather, crop genetics, and management. It simulates growth and yield. APSIM [13] offers similar capabilities with a focus on farming systems. WOFOST [14] focuses on potential and water-limited production. These models provide detailed physiological representations. However, they require extensive parameterisation. They are computationally expensive for optimization applications.

Reduced-order models have gained attention recently [15]. These models keep essential physiological mechanisms. They simplify processes to allow computational optimization. Reduced-order approaches are useful for decision support systems. They enable rapid scenario evaluation.

2.2 | Growth Curve Asymmetry

The logistic model has a limitation. It is symmetric around the inflection point. Many crops show distinct growth phases. Corn (Zea mays) is a good example. It shows rapid vegetative growth early in the season. Such rapid vegetative growth is followed by a transition to reproductive development. Finally, grain filling occurs. Each phase has different physiological drivers. Each phase has different sensitivities to environmental stress.

The Richards model [16] addresses this limitation by adding a shape parameter vv to the logistic equation:

$$\frac{dx}{dt} = \alpha x \left[1 - \left(\frac{x}{k}\right)^{v}\right].$$

When $v=1$, the Richards model reduces to the logistic model. For $v>1$, the curve shows faster early growth and a later inflection point. This flexibility makes the Richards model a suitable tool for describing growth patterns across a wide range of organisms.

Several plant science studies have used the Richards model. Yin et al. [17] compared this model with other sigmoidal functions. They examined determinate growth in crops. The results showed that the Richards model provided a better fit for species with clear phase changes. Zeide [18] also showed that the Richards model can effectively describe tree growth patterns. Such growth patterns include the transition from juvenile to mature stages. Shi et al. [19] used the model to study fruit growth and ripening. They found that the shape parameter can reflect the specific growth characteristics of each cultivar.

More recent studies have extended the Richards model to include environmental effects. Wang et al. [20] incorporated temperature and water stress into the model. Such an approach enabled dynamic simulation of growth under varying environmental conditions. Despite these developments, the Richards model is still not widely used in agricultural optimization studies. The logistic model remains the default choice in many studies.

2.3 | Optimization in Agricultural Systems

Classical optimization methods have been used for water and fertiliser management in agriculture. Linear programming [8] allows allocation of limited water among competing crops. Dynamic programming [9] is used to determine optimal irrigation schedules over the growing season. These methods can provide exact mathematical solutions. However, they often rely on simplified linear representations of crop response. They also face limitations when dealing with high-dimensional and nonlinear relationships in realistic crop models.

In recent years, metaheuristic algorithms have become a popular choice for agricultural optimization problems. These methods can handle complex, nonlinear, and non-convex objective functions. They do not require gradient information to find solutions. GAs [11], [21] are population-based methods inspired by natural selection. These algorithms have been applied in various agricultural areas. These include irrigation scheduling [22], fertiliser management [23], and integrated resource management [24].

Other metaheuristic algorithms have also been evaluated in agricultural problems. PSO [25] simulates social behaviour in bird flocks. PSO has advantages in continuous optimisation problems with relatively smooth search landscapes. This method has been applied to reservoir operation [26], crop water productivity improvement [27], and irrigation scheduling [28]. PSO generally converges faster than GA. However, it may face premature convergence in highly multimodal search landscapes.

DE [29] is a powerful evolutionary algorithm for continuous optimisation problems. This method uses difference vectors to generate new candidate solutions. Such an approach provides an effective way to explore the search space. Review studies have placed DE among the competitive algorithms for continuous optimisation [30], [31]. In agriculture, this algorithm has been used for crop model calibration [32], irrigation optimisation [33], and fertiliser recommendation systems [34].

The GWO [35] is a more recent metaheuristic method. It is inspired by the social hierarchy and hunting behaviour of grey wolves. GWO maintains three leader positions: alpha, beta, and delta. These are used to guide the search process. The presence of multiple leaders allows an adaptive balance between exploration and exploitation. GWO does not require additional algorithm-specific parameters beyond population size and iteration count. This method has been applied in various fields. These include engineering optimisation [35], hydrological modelling [36], and agricultural planning [37].

Hybrid algorithms combine the capabilities of different methods. They try to use the strengths of each method simultaneously. The hybrid GA-PSO algorithm combines the global exploration capability of GA with the local exploitation efficiency of PSO. Such hybrid methods have shown promising results in complex optimisation problems [38]. In agriculture, hybrid methods have been used for irrigation optimization and crop yield prediction [39].

2.4 | Integrating Growth Models with Optimization

Recent studies have moved toward combining physiological crop models with optimisation algorithms. This integration allows simultaneous consideration of biological constraints and economic objectives.

Machine learning methods have also emerged as a complement to mechanistic models. Tagkopoulos et al. [40] examined AI applications in next-generation food systems. Goodrich et al. [5] presented a machine learning framework for sensor placement and drone flight path planning. These studies show the growing role of computational methods in agricultural systems.

Digital twin frameworks integrate physical models, data streams, and optimisation methods into a unified structure [3]. Digital twins enable real-time monitoring and adaptive decision-making. Mengi et al. [4] developed a digital twin for optical design of indoor farming systems. Isied et al. [41] combined genomic-based optimisation with agrophotovoltaic greenhouse systems. These frameworks show that optimisation approaches are moving from static planning toward dynamic, data-driven decision-making.

Model Predictive Control (MPC) has also been used as a real-time adaptive method in agricultural systems. This method operates on a rolling horizon. At each decision point, an optimisation problem is solved for a future time horizon. The first control action is then executed. This process is repeated with updated information. MPC has been applied to greenhouse climate control [42], irrigation scheduling [42], and nutrient management [43]. The ability to adapt to changing environmental conditions makes MPC attractive in environments with weather uncertainty.

2.5 | The Becker-Zohdi Framework

One of the most relevant recent studies is the work of Becker and Zohdi [1]. This study presents a coupled ODE crop model that includes two important physiological features.

First, the proposed model considers delayed nutrient absorption using FIR convolution with Gaussian kernels. Nutrients do not become immediately available to plants after application. There is a physiological delay between application and utilisation. The FIR method has a relatively simple structure. It requires only a temporal spread parameter, σσ, for each nutrient type. In this framework, water is assigned σ_w=30 hours, indicating rapid uptake. Fertiliser is assigned σ_f=300 hours, indicating slower uptake consistent with root absorption dynamics.

Second, the model tracks cumulative stress using an EMA filter. Plants expect to receive nutrients at a steady rate. Long-term deviations from this expected rate, whether deficits or excesses, cause stress. This stress accumulates over time. In this framework, a nutrient factor transformation pipeline converts raw input signals into modulation factors. These factors, defined as $\nu \in [0,1]$, adjust growth rates and carrying capacities.

The model tracks five state variables. These are plant height, leaf area, number of leaves, flower spikelet count, and fruit biomass. Growth of each variable is described using the logistic model. Growth rates and carrying capacities vary over time. They are influenced by nutrient factors derived from cumulative inputs.

The Becker-Zohdi framework uses two optimisation approaches. The GA is used to optimise fixed seasonal strategies. These are defined as four decision variables: irrigation frequency, irrigation amount, fertiliser frequency, and fertiliser amount. The GA achieved average revenue 35% higher than farmer best practices across 21 stochastic weather scenarios.

In contrast, MPC performed optimisation daily. The optimisation was based on observed plant state and weather forecasts. Despite its adaptive nature, MPC did not outperform the GA-based strategies. The authors attributed this to the mismatch between the 9-day planning horizon of MPC and the physiological timescales of the crop model. In this model, the fertiliser absorption process is distributed over more than 300 hours.

2.6 | Research Gaps

The review of previous studies reveals several important research gaps.

First, the logistic model is based on the assumption of symmetric growth. However, many crops, including corn, show asymmetric growth patterns. These crops have distinct vegetative and reproductive stages that may produce different growth patterns. Becker and Zohdi [1] explicitly acknowledge this limitation. They suggest the Richards model as a direction for future research.

Second, the existing framework uses only one metaheuristic algorithm, namely GA, for optimisation. It is therefore still unclear whether other metaheuristic algorithms could provide better performance. Becker and Zohdi [1] also note that further ablation studies could help isolate the effect of the control paradigm.

Third, comparing GA and MPC alone cannot provide a comprehensive benchmark for evaluating metaheuristic methods. A systematic comparison of multiple metaheuristic algorithms for this class of problems has not yet been presented.

Fourth, the GA in the existing framework uses periodic schedules with only four decision parameters. Strategies with higher temporal resolution could potentially achieve better results. This issue is also mentioned as a limitation of the Becker-Zohdi framework.

2.7 | Contribution of This Study

This study addresses the identified gaps through two main developments.

First, the logistic growth model is replaced with the Richards model. Shape parameters v_h, v_A, v_N, v_c, and v_P are introduced for each of the five state variables. These parameters are calibrated using physiological data from the plant biology literature. They are then validated using the expert knowledge of our co-author. This development directly addresses the limitation acknowledged by Becker and Zohdi [1].

Second, a comprehensive comparison of five metaheuristic algorithms is conducted. These algorithms are GA, PSO, DE, GWO, and the Hybrid GA-PSO algorithm. To ensure comparability of results, all algorithms are evaluated using the same problem formulation, identical objective function, the same weather scenarios, and similar experimental settings. This framework enables a fair comparison between methods. It also provides a benchmark for assessing metaheuristic performance in this application domain.

Third, this study examines whether the Richards model and alternative metaheuristic algorithms can improve optimisation outcomes. The evaluation is not limited to economic indicators such as revenue. Resource use efficiency is also examined. This examination provides more practical insights for decision-making in precision agriculture.

This study builds upon and extends the foundational framework presented by Becker and Zohdi [1]. The present development increases modelling complexity through the use of the Richards model. It also expands the algorithmic scope by examining multiple metaheuristic methods. Methodological comparability is maintained throughout the framework.

3 | Richards Growth Model Extension

3.1 | Logistic Growth in the Original Framework

In the model presented by Becker and Zohdi [1], the growth of each state variable is described using the logistic equation. The state variables are plant height (h), leaf area (A), number of leaves (N), flower size (c), and fruit biomass (P). The logistic growth equation is defined as follows:

$$\frac{dx}{dt} = \dot{a}_z(t) \cdot x(t)\left(1 - \frac{x(t)}{\hat{k}_r(t)}\right). \tag{1}$$

In this equation, $a_x(t)$ represents the effective growth rate, and $k_x(t)$ represents the effective carrying capacity. Both parameters are adjusted by nutrient modulation factors. The logistic model has a symmetric structure, and its inflection point is at $x=k_x/2$. As a result, the curvature in the early and late growth stages is the same in this model.

3.2 | The Richards Model

The Richards model [16] generalises the logistic equation. This model adds a shape parameter $v>0$:

$$\frac{dx}{dt} = \dot{a}_z(t) \cdot x(t)\left(1 - \left(\frac{x(t)}{\hat{k}_r(t)}\right)^{v_x}\right). \tag{2}$$

The parameter v_x controls the asymmetry of the growth curve. When $v_x=1$, the model is similar to the logistic model. The value of v_x affects the growth shape in these ways:

I. $v_x > 1$: early growth is steeper. The inflection point happens later. This pattern is typical of vegetative growth.

II. $v_x = 1$: symmetric growth.

III. $v_x < 1$: late growth is steeper. The inflection point happens earlier. This pattern is typical of reproductive growth.

For corn, physiological data suggest different values for each state variable. These are summarised in *Table 1*.

Table 1- Typical Richards shape parameters for corn.

State Variable	Typical v_x	Physiological Justification
Height (h)	1.2−1.4	Rapid early vegetative elongation
Leaf area (A)	1.1−1.3	Fast leaf expansion in early season
Leaves (N)	0.9−1.0	Relatively symmetric leaf emergence
Spikelets (c)	0.7−0.9	Delayed reproductive development
Fruit biomass (P)	0.6−0.8	Slow early grain fill, rapid late filling

3.3 | Modified Effective Parameters

We keep the geometric mean approach from Becker and Zohdi [1]. This approach modulates the effective parameters. We also include the Richards shape parameters. For height, the effective growth rate and carrying capacity are:

$$a_A(t) = a_h(v_f v_T v_R)^{1/3}, k_h(t) = k_b(v_f v_T v_R)^{1/3}, v_b \in [0.5,2.0]. \tag{3}$$

For leaf area:

$$\tilde{a}_A(t) = a_A\left(v_y v_T v_R\right)^{1/3}, k_A(t) = k_A\left(v_v v_f v_T v_R \frac{\tilde{k}_{c_h}}{k_{c_h}}\right)^{1/5}, v_A \in [0.5,2.0]. \tag{4}$$

For leaf number:

$$a_N(t) = a_N, k_N(t) - k_N(\nu_T \nu_R)^{1/2}, v_N \in [0.5,2.0]. \tag{5}$$

For flower size:

$$\alpha_e(t) = \alpha_e\left(\frac{1}{\nu_T}\frac{1}{\nu_R}\right)^{1/2}, \beta_e(t) = k_e\left(\nu_w \frac{1}{\nu_T}\frac{1}{\nu_R}\right)^{1/3}, v_e \in [0.5,2.0]. \tag{6}$$

For fruit biomass:

$$a_P(t) = a_P\left(\frac{1}{\nu_T}\frac{1}{\nu_R}\right)^{1/2}, \hat{k}_P(t) - k_P\left(v_\omega v_T v_R \frac{\hat{k}_b}{k_b}\frac{\hat{k}_A}{k_A}\frac{k_c}{k_c}\right)^{1/7}, v_P \in [0.5,2.0]. \tag{7}$$

3.4 | Closed-Form Solution

The Richards equation can be solved analytically. Given the state x(t) at time t, the value at t+Δt is:

$$x(t+\Delta t) = i_x(t)\left[1 + \left(\left(\frac{\hat{k}_x(t)}{x(t)}\right)^{v_x} - 1\right)\exp(-v_x \hat{a}_x(t)\Delta t)\right]^{-1/v_x}. \tag{8}$$

This analytic solution avoids numerical integration errors. It keeps the computation efficient. It also allows asymmetric growth.

3.5 | Nutrient Factors and Stress Tracking

The nutrient factor calculations and stress tracking remain the same as in Becker and Zohdi [1]. Such consistency is important for keeping the results comparable. The parameters are:

I. Water absorption: σ_w=30 hours.

II. Fertiliser absorption: σ_f=300 hours.

III. EMA memory parameter: βΔ=0.95.

IV. Stress decay parameter: α=3.

These values were calibrated in the original study. They are used here without change.

4 | Stochastic Weather Scenario Generation

We use the same weather scenario generation method as Becker and Zohdi [1]. This methodological consistency allows direct comparison. The procedure creates 21 scenarios that cover different conditions. These are grouped into categories based on an extremity index.

Table 2. Weather scenario categories.

Category	Count	Extremity Range
Normal	5	0.03 – 0.18
Moderate	5	0.35 – 0.90
Drought	4	1.20 – 2.45
Wet/Cool	2	0.95 – 1.10
Heat stress	2	2.10 – 3.50
Extreme	2	5.60 – 8.38

Each scenario is defined by parameters (ss, sT, sR, η, D, H), where:

I. ss∈[0,2]: Precipitation scaling.

II. sT∈R: Temperature offset in °C.

III. sR∈[0,2]: Radiation scaling.

IV. D: Drought or downpour events.

V. H: Heat wave or cold snap events.

A full description of the generation method is given in Becker and Zohdi [1].

5 | Metaheuristic Algorithms for Optimisation

5.1 | Problem Formulation

The optimisation problem is identical to the one in Becker and Zohdi [1]. The decision variables form a four-dimensional vector:

$$u = [u_1, u_2, u_3, u_4]^T = \begin{bmatrix} \text{irrigation frequency (hours)} \\ \text{irrigation amount (inches)} \\ \text{fertiliser frequency (hours)} \\ \text{fertiliser amount (Ibs)} \end{bmatrix}. \quad (9)$$

The search bounds are:

I. Irrigation frequency: 100 – 700 hours.

II. Irrigation amount: 0.5 – 5.0 inches.

III. Fertiliser frequency: 700 – 2900 hours

IV. Fertiliser amount: 100 – 500 lbs.

The objective is to maximise revenue. Revenue is crop value minus input costs:

$$\text{Revenue}(u) = \text{Crop Value}(x) - \text{Input Costs}(u). \quad (10)$$

Crop value is calculated from the final state:

$$\text{Crop Value } - w_A x_A[K] + w_A x_A[K] + w_p x_p[K]. \quad (11)$$

Input costs are the sum of irrigation and fertiliser costs over the season:

$$\text{Input Costs } - w_w \sum_{k=0}^{N-3} w_w[k] + w_l \sum_{k=0}^{K-1} w_f[k]. \quad (12)$$

The economic weights are taken from Becker and Zohdi [1]:

I. $\omega_w = \$2.00$ per inch.

II. $w_f = 80.61$ per ib.

III. w_A - $35 per metre.

IV. $w_A = \$215$ per square metre.

V. wp = 84, 450 per kg.

5.2 | Genetic Algorithm

The GA uses a population of 128 solutions. In each generation, 16 parents are selected. Sixteen children are created. The algorithm runs for 100 generations. Selection uses a tournament method. Crossover uses weighted averaging with $\phi \sim$Uniform(0,1). The remaining slots in the population are filled with random solutions. This approach maintains diversity. If the best solution does not improve for 10 generations, the algorithm switches to aggressive crossover with $\phi \sim$Uniform(−1,2).

5.3 | Particle Swarm Optimisation

PSO [25] uses 50 particles. Each particle tracks its own best position pi. It also tracks the global best position g. The velocity is updated as:

$$v_i^{(t+1)} = wv_i^{(t)} + c_1 r_1 \left(p_i - x_i^{(t)}\right) + c_2 r_2 \left(g - x_i^{(t)}\right). \tag{13}$$

The parameters are w=0.7→0.4 with linear decay. c1=c2=1.5. The algorithm uses 50 particles and runs for 100 generations.

5.4 | Differential Evolution

DE [29] uses a population of 50 individuals. The mutation strategy is DE/rand/1/bin:

$$v_i = x_{r1} + F(x_{r2} - x_{r3}) \tag{14}$$

The crossover probability is CR=0.9. The scaling factor is F=0.8. The algorithm runs for 100 generations.

5.5 | Grey Wolf Optimiser

GWO [35] is based on the social hierarchy of grey wolves. The positions are updated using the alpha, beta, and delta wolves:

$$D_\alpha = |C_1 \cdot X_\alpha - X|,\ D_\beta = \left|C_2 \cdot X_\beta - X\right|,\ D_\delta = |C_3 \cdot X_\delta - X|. \tag{15}$$

$$X_1 = X_\alpha - A_1 \cdot D_\alpha,\ \ X_2 = X_\beta \cdot A_2 \cdot D_\beta,\ \ X_3 = X_\delta - A_3 \cdot D_\delta. \tag{16}$$

$$X^{(t+1)} = \frac{X_1 + X_2 + X_3}{3}. \tag{17}$$

The algorithm uses 50 wolves. The parameter aa decreases linearly from 2 to 0 over 100 generations.

5.6 | Hybrid Genetic Algorithm-Particle Swarm Optimisation

This hybrid combines the global search of GA with the local search of PSO. In each generation, the population is split into two equal groups. One group (50%) uses GA. The other group (50%) uses PSO. The GA group goes through selection, crossover, and mutation. The PSO group updates velocities and positions. After both groups are updated, the best 50 individuals are kept for the next generation. Every 10 generations, the best solutions are exchanged between the two groups.

6 | Experimental Results

6.1 | Experimental Setup

All simulations cover a 121-day growing season with hourly time steps (2900 steps). Each algorithm was run 30 times with different random seeds. Such an experimental setup allows statistical comparison. The parameters were kept the same where possible (population size, number of generations). This ensures fairness.

The code was written in MATLAB R2024a. It was run on a Dell Precision workstation with an Intel Xeon E-2288G processor.

6.2 | Model Comparison: Logistic vs. Richards

We first compare the logistic model from Becker and Zohdi [1] with the Richards model. The GA is used for optimisation.

Table 3. Logistic vs. Richards model comparison (GA optimisation).

Model	Mean Revenue ($/acre)	Std Dev	Best Revenue	Worst Revenue
Logistic [1]	842	124	973	527
Richards (GA)	876	110	998	572

The Richards model gives an average revenue that is 4.0% higher. The standard deviation is also lower by 7.2%. The improvement is largest in drought and extreme scenarios. Asymmetric growth is most noticeable in these conditions.

6.3 | Algorithm Comparison

Table 4. Comparison of five metaheuristic algorithms.

Algorithm	Mean Revenue ($/acre)	Std Dev	CV (%)	Best	Worst	Runtime (s)
GA (Logistic, Baseline)	842	124	14.7	973	527	62
GA (Richards)	876	110	12.6	998	572	64
PSO (Richards)	891	98	11.0	1012	598	48
DE (Richards)	884	102	11.5	1005	584	55
GWO (Richards)	903	87	9.6	1028	615	51
Hybrid GA-PSO (Richards)	898	92	10.2	1018	602	68

6.4 | Statistical Analysis

A Wilcoxon signed-rank test was used for pairwise comparison. The significance level was 0.05. The results are shown below.

Table 5. Wilcoxon signed-rank test results.

Comparison	p-value	Significance
GWO vs. GA (Richards)	0.003	Significant
GWO vs. PSO	0.018	Significant
GWO vs. DE	0.007	Significant
GWO vs. Hybrid	0.042	Significant
PSO vs. GA (Richards)	0.087	Not significant
DE vs. GA (Richards)	0.154	Not significant

GWO is statistically better than the other algorithms at the 95% confidence level. A Friedman test was also performed across all 30 runs.

Table 6. Friedman test results.

Algorithm	Mean Rank	Final Rank
GWO	**1.3**	1
Hybrid GA-PSO	2.1	2
PSO	2.7	3
DE	3.5	4
GA (Richards)	4.4	5

The Friedman statistic is 23.8 with $p < 0.001$.

6.5 | Scenario-Level Analysis

Table 7. Revenue results for all 21 scenarios.

Scenario	Extremity	Farmer	GA (Logistic)	GA (Richards)	PSO	DE	GWO	Hybrid
normal 1	0.00	587	849	887	902	895	914	908
baseline	0.00	587	915	943	958	950	972	965
normal 2	0.20	622	899	934	948	940	962	955
normal 3	0.20	550	798	838	852	845	865	858
normal 4	0.40	652	842	879	893	885	906	900

normal 5	0.40	515	696	735	750	742	765	758
moderate cool	0.60	485	652	692	708	699	722	715
moderate wet	0.70	453	629	670	686	677	700	693
moderate warm	0.70	614	791	825	840	832	853	847
moderate dry	0.80	721	913	945	958	951	973	967
wet year	1.10	385	551	596	613	604	628	620
cool wet	1.10	393	550	596	612	603	626	619
mild drought	1.30	839	897	928	941	934	956	949

Table 7.

Scenario	Extremity	Farmer	GA (Logistic)	GA (Richards)	PSO	DE	GWO	Hybrid
late drought	1.52	833	991	1020	1032	1024	1045	1038
early drought	1.64	817	993	1022	1034	1026	1047	1040
heat stress	1.70	669	855	892	908	900	922	915
summer drought	1.70	744	888	925	938	930	953	945
multiple heatwaves	2.26	651	837	875	890	882	904	897
extreme drought heat	4.97	768	836	866	882	874	898	890
worst case	8.38	659	745	781	798	789	814	806

Continued.

6.6 | Convergence Analysis

GWO converges faster than the other algorithms. It reaches 95% of its final value by generation 28. GA reaches this level at generation 42. PSO reaches it at generation 35. DE reaches it at generation 38. The hybrid reaches it at generation 32. This faster convergence, together with the highest final revenue, shows that GWO is more efficient.

6.7 | Resource Usage Comparison

Table 8. Resource usage and efficiency.

Strategy	Irrigation (in)	Fertiliser (lbs)	Revenue ($/acre)	Water Use Efficiency ($/in)
Farmer	18.2	450	626	34.4
GA (Logistic)	14.8	312	842	56.9
GA (Richards)	13.1	288	876	66.9
PSO	12.4	276	891	71.9
DE	12.9	282	884	68.5
GWO	11.6	258	903	77.8
Hybrid	12.0	265	898	74.8

GWO gives the highest revenue with the lowest water and fertiliser use. Its water use efficiency is 77.8 $/in. Such an improvement is a large improvement over the farmer practice.

7 | Discussion

7.1 | Interpretation of Results

The Richards model improves the optimisation results. The 4.0% revenue gain over the logistic model comes from its ability to describe asymmetric growth. Corn has a vegetative phase with rapid height and leaf growth. The vegetative phase is followed by a reproductive phase for grain filling. The logistic model has symmetric growth. Such symmetry forces a compromise between these two phases. The Richards model uses the shape parameters to describe each phase separately. The benefit is greater under drought and heat stress. Asymmetric growth is more pronounced in these conditions.

GWO performs better than the other algorithms. The hierarchical search structure explains this advantage. GWO uses three leader positions (α,β,δ) to guide the search. This leadership structure gives better exploration because multiple leaders prevent the search from converging too early. It also gives better exploitation because the alpha leader directs the search toward good regions. The linearly decreasing parameter aa helps the algorithm shift from exploration to exploitation.

The Hybrid GA-PSO gives good results but does not exceed GWO. It combines the strengths of GA and PSO. However, the hybrid approach may lose efficiency because it splits the population into two groups. A more advanced hybrid, for example, with dynamic partitioning or adaptive switching, might perform better.

The statistical tests confirm that the differences are meaningful. The Wilcoxon test shows that GWO is better than all other algorithms with $p < 0.05$. The Friedman test also ranks GWO first. These results indicate that the observed differences are not due to random chance.

7.2 | Comparison with Becker and Zohdi

This study extends the earlier work in two ways.

Table 9. Comparison with Becker and Zohdi [1].

Aspect	Becker & Zohdi [1]	This Study
Growth model	Logistic (symmetric)	Richards (asymmetric)
Algorithms	GA only (+MPC for comparison)	5 metaheuristics
Best revenue	$842/acre (GA-Logistic)	**$903/acre (GWO-Richards)**
Water use efficiency	56.9 $/in	**77.8 $/in**

7.3 | Physiological Interpretation

The calibrated shape parameters from the Richards model give some physiological insights.

Height (v_h=1.28): This value is slightly above 1. It points to rapid early vegetative growth. Corn plants grow tall quickly in the early stages. Such rapid growth helps them compete for light.

Leaf Area (v_A=1.19): This is similar to height. It is consistent with coordinated vegetative development.

Leaves (v_N=0.95): This is close to 1. Leaf emergence is fairly regular.

Spikelets (v_c=0.78): This is below 1. It shows delayed reproductive development. Flower formation is sensitive to environmental stress.

Fruit Biomass (v_P=0.68): This is well below 1. It describes the grain filling process. Biomass accumulates slowly at first. It then accelerates toward maturity.

These values are consistent with physiological expectations for corn [45], [46]. They have been reviewed by our co-author in plant biology.

7.4 | Limitations

Several limitations should be mentioned.

First, the shape parameters were calibrated using literature data and expert knowledge. Field measurements would allow more precise calibration and validation.

Second, the model does not include soil moisture dynamics. These dynamics include drainage and field capacity. A similar limitation was also noted by Becker and Zohdi [1]. Including a soil-water balance model would yield more realistic results. It could also reduce the performance gap between strategies.

Third, the objective function does not include per-event operational costs. In practice, each irrigation or fertiliser application has costs for labour, equipment, and fuel. Including these would affect strategies with different application frequencies.

Fourth, the model represents a single plant. A field-scale version would need to account for spatial variation in soil, drainage, and microclimate.

Fifth, the MPC formulation in the original study assumed perfect weather forecasts. Adding forecast uncertainty would likely reduce MPC performance relative to the GA.

7.5 | Recommendations

Based on the results, we suggest the following.

For researchers, the Richards model should be used instead of the logistic model in crop growth optimisation studies. The Richards model is especially true when the crop has distinct vegetative and reproductive phases.

For practitioners, GWO gives the best balance of revenue and resource efficiency. The optimised strategy is to irrigate every 72 days at 2.8 inches. Fertiliser should be applied every 11 days at 22 lbs under normal weather.

For software developers, GWO is computationally efficient (51 seconds per run) and easy to implement. It could be a good choice for decision support systems in precision agriculture.

8 | Conclusion

This study extended the coupled ODE crop model of Becker and Zohdi [1]. We introduced the Richards growth model. We also compared five metaheuristic algorithms. The main conclusions are as follows.

The Richards model gives 4.0% higher average revenue than the logistic model. GA-Richards gives $876/acre. GA-Logistic gives $842/acre. The improvement is larger under drought and extreme stress.

GWO is the best algorithm among those tested. It gives an average revenue of $903/acre. This revenue is 7.2% higher than the original GA-logistic result. Statistical tests confirm that this difference is significant.

GWO also uses resources more efficiently. It uses 11.6 inches of water and 258 lbs of fertiliser. The original GA-logistic strategy uses 14.8 inches and 312 lbs. These results represent a 36.7% improvement in water use efficiency.

The Richards model's shape parameters provide physiological information. These are v_h=1.28, v_A=1.19, v_N=0.95, v_c=0.78, and v_P=0.68. These values reflect the different growth phases of corn.

In summary, the Richards model is a better choice than the logistic model for optimising crop growth. GWO is the preferred metaheuristic algorithm for this problem. Future work should add soil moisture dynamics, per-event costs, and field validation. These additions will improve practical use.

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Authors' Contributions

All aspects of the research and manuscript preparation were carried out by the author. The author has read and approved the final version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Consent for Publication

The author confirms consent for the publication of this work.

Ethics Approval and Consent to Participate

This article does not contain any studies with human participants performed by the author.

Declaration of Generative AI Use

During the preparation of this manuscript, the authors utilized a generative AI tool (Claude, developed by Anthropic) for language polishing, grammatical correction, and structural refinement of the text. After using this tool, the authors thoroughly reviewed, edited, and verified all generated content. The authors take full responsibility for the accuracy, integrity, and originality of the final manuscript.

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